Math 266 **Midterm 1 Study Guide** **Spring 2019**

Terminology you should know:

• differential equation

• direction field, slope field (same thing as direction field), equilibrium solution

• initial condition, initial value problem

• solution to a differential equation, general solution, solution to an initial value problem, integral curve

• linear differential equation, nonlinear differential equation, order of a differential equation

• integrating factor

• separable equation, homogeneous equation (y/x version), Bernoulli equation, autonomous equation, exact equation

• explicit solution, implicit solution, interval where the solution is valid

• mathematical model, gravity, air resistance, concentration (of some chemical in another chemical), etc.

• existence of a solution, uniqueness of a solution

• equilibrium solution, critical point, asymptotically stable, unstable, semistable, $f(y) vs. y$ graph, phase line, carrying capacity, threshold

• Euler's method

Things you should know how to do:

• Know how to graph a direction field, look at a direction field and describe behavior of solutions, identify which differential equation from a list produces a given direction field
Practice Problems:

Consider the following list of differential equations, some of which produced the direction fields shown in Figures 1.1.5 through 1.1.10. In each of Problems 15 through 20 identify the differential equation that corresponds to the given direction field.

(a) $y' = 2y - 1$ (b) $y' = 2 + y$ (c) $y' = y - 2$
(d) $y' = y(y + 3)$ (e) $y' = y(y - 3)$ (f) $y' = 1 + 2y$
(g) $y' = -2 - y$ (h) $y' = y(3 - y)$ (i) $y' = 1 - 2y$
(j) $y' = 2 - y$

15. The direction field of Figure 1.1.5.
16. The direction field of Figure 1.1.6.

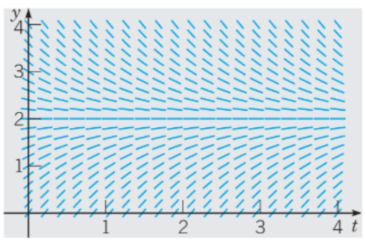

FIGURE 1.1.5 Problem 15.

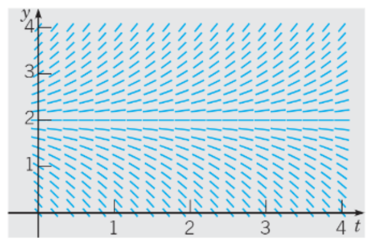

FIGURE 1.1.6 Problem 16.

17. The direction field of Figure 1.1.7.
18. The direction field of Figure 1.1.8.

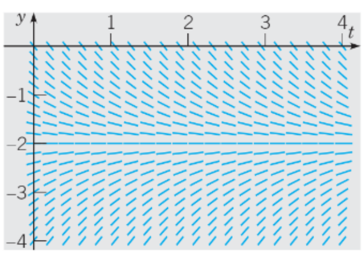

FIGURE 1.1.7 Problem 17.

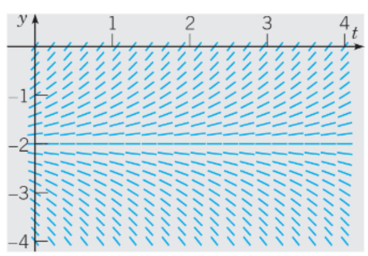

FIGURE 1.1.8 Problem 18.

19. The direction field of Figure 1.1.9.
20. The direction field of Figure 1.1.10.

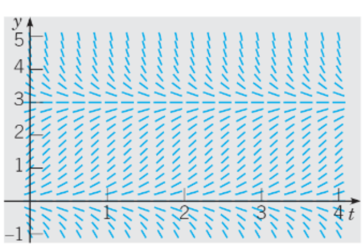

FIGURE 1.1.9 Problem 19.

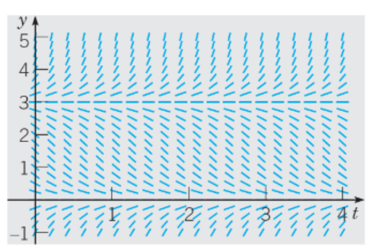

FIGURE 1.1.10 Problem 20.

• Identify a differential equation as ordinary vs. partial, linear vs. nonlinear, and the order of the differential equation

<u>Practice Problems</u>: For each of the differential equations below, determine whether it is partial or ordinary, linear or nonlinear, and state the order.

$$t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+2y=\sin t$$
$$\frac{d^2y}{dt^2}+\sin(t+y)=\sin t$$
$$\frac{d^3y}{dt^3}+t\frac{dy}{dt}+\left(\cos^2 t\right)y=t^3$$
$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$$

• Given a function, identify whether or not it is a solution to a given differential equation

<u>Practice Problems</u>: For what value(s) of A, if any, will y $= Ate^{-2t}$ be a solution of the differential equation $2y'+4y=3e^{-2t}$? For what value(s) of B, if any, will y $= Be^{-2t}$ be a solution?

• Know when to use the integrating factor method, compute the integrating factor $\mu(t)$, solve a differential equation with the integrating factor method

<u>Practice Problems</u>: Find the solution to the IVP $t^3y'+4t^2y=e^{-t}, y(-1)=0, t<0$.

• Identify an equation as separable, solve a separable equation implicitly and explicitly (if possible), state where the solution to such an equation is valid

<u>Practice Problems</u>: Solve the IVP

$$\frac{dy}{dx}=\frac{x^2}{y}, y(0)=-\frac{2}{3}$$

Determine the interval on which the solution is valid.

• Identify an equation as homogeneous (y/x) and solve a homogeneous equation with the substitution $u(x)=y/x$, know that after a substitution the result is always separable, know about the symmetry of the direction field of a homogeneous equation

<u>Practice Problems</u>: Show the following differential equation is homogeneous and find a solution:

$$\frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$$

• Be able to solve a differential equation given a substitution (see supplementary problems B and C)

• Model tank problems, model falling body problems, model Newton's Law of Cooling problems (see HW 6 and 7)

• Understand and be able to apply Theorems 2.4.1 and 2.4.2 about the existence and uniqueness of solutions to initial value problems.

Practice Problems: Find the largest interval on which the solution to the IVP is guaranteed to exist

$$(t-3)\,y' + (\ln t)\,y = 2t, y(1) = 2$$

• Understand the limitations of Theorem 2.4.2 when compared to Theorem 2.4.1

Practice Problems: Even though the hypotheses of Theorem 2.4.2 are satisfied on the entire ty-plane for $y' = y^2 + 6y, y(0) = 2$, show (by finding the solution) that the domain is smaller than the interval $-\infty < t < \infty$.

• Identify a Bernoulli equation and solve a Bernoulli equation making the appropriate substitution, know that Bernoulli equations are nonlinear but after the appropriate substitution become linear.

Practice Problems: Solve $t^2y' + 2ty - y^3 = 0, t > 0$

• Given an autonomous equation, graph f(y) vs. y, graph the phase line, find equilibrium solutions, classify equilibrium solutions as asymptotically stable, unstable, or semistable, draw a qualitatively accurate sketch of several integral curves based on the phase line and equilibrium solutions, be able to talk about the carrying capacity and threshold of a population which can be modeled by an autonomous equation.

Practice Problems: Draw the phase plane and several qualitatively accurate sketches of solutions for the differential equation

$$\frac{dy}{dt} = y\left(1 - y^2\right)$$

Determine the equilibrium solutions and state whether each is asymptotically stable, unstable, or semistable.

Suppose a population is modeled by

$$\frac{dp}{dt} = p^2\,(5-p)\,(p-12)$$

where p is measured in thousands. What are the threshold and carrying capacity of the population?

• Show that a differential equation is exact, find the function $\psi(x,y)$ associated with an exact equation, solve an exact equation

Practice Problems: Show the following diff eq is exact and find an implicit solution

$$y' = \frac{6y+2x}{3y^2-6x}$$

• Understand how Euler's method works, use Euler's method to do some simple estimations of solutions to initial value problems

• Describe the behavior of solutions to a differential equation as $t \to \infty$, finding maxima and minima of solutions, finding critical initial values (values of a so that if $y(t_0) < a$, then the solution behaves in a very different way from if $y(t_0) > a$)

Practice Problems: Find the critical value a_0 of a for which behavior changes qualitatively. Then describe the behavior of solutions to the following IVP as $t \to \infty$ (depending on the value of a).

$$y' - \frac{1}{2}y = 2\cos(t)\,,\, y(0) = a$$